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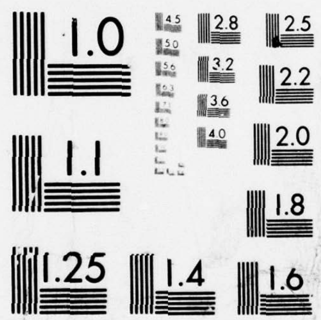
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Linear Stability of Self-Similar Flow: 4. Convective Instability of a Spherical Cloud Expanding into Vacuum

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The well-known class of self-similar solutions for an ideal polytropic gas sphere of radius $R(t)$ expanding into a vacuum with velocity $u(r,t) = r\dot{R}/R$ is shown to be convectively unstable. The physical mechanism results from the buoyancy force experienced by anisentropic distributions in the inertial gravitational field. An equation for the perturbed displacement $\cancel{\xi(r,t)}$, derived from the linearized fluid equations in Lagrangian coordinates, is solved by separation of variables. Because the basic state is nonsteady, the perturbations do not grow exponentially, but can be expressed in terms of hypergeometric functions. For initial density

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profiles $\rho_o(r) \sim (1-r^2/r_o^2)^{\kappa}$, modes with angular dependence $Y_{\ell m}(\theta,\phi)$ are unstable provided $\ell > 0$ and $\kappa < 1/(\gamma-1)$, where γ is the ratio of specific heats. For large ℓ, the characteristic growth time of the perturbations varies as $\ell^{-1/2}$ and the amplification increases exponentially as a function of ℓ. The radial eigenfunctions are proportional to r^{ℓ}, and the compressibility and vorticity are both non-zero.

I. Introduction

Over a quarter century has passed since the discovery that the equations of one-dimensional ideal fluid motion possess a variety of self-similar solutions. These were found independently and more or less simultaneously by Sedov (1953), Staniukovich (1949), Taylor (1950) and others, stimulated by interest in nuclear and astrophysical explosions and in the general properties of gas dynamic systems. The usefulness of these solutions is twofold: they correctly describe 1-D flows at late times when the details of initiation or preparation have been "forgotten," and they are analytic, or at least reduce the solution to quadratures.

A particularly useful and interesting type of self-similar motion is that known as uniform or homogeneous. Its characteristic feature is a velocity field which is proportional to the distance from the center of symmetry. Most applications have been to problems with spherical symmetry, such as supernova explosions (Keller, 1956), laser implosions (Kidder, 1976) and self-gravitating clouds (Sedov, 1959). Closely related to the latter are cosmological models in the nonrelativistic limit (Weinberg, 1972), which are distinctive by virtue of being pressureless and unbounded.

Sedov (1959) distinguishes three types of uniform self-similar motion in an ideal gas. In type I, the radius varies between 0 and ∞. In type II, it varies between 0 and a finite maximum value, corresponding to a turning point. Type III, to which we restrict ourselves in the present work, has radius varying between a minimum (at a turning

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point) and ∞. We follow Keller (1956) in using Lagrangian coordinates to derive in §2.1 a two-parameter family of solutions including, among others, isothermal and uniform-density models as special cases. There seems at first glance no reason why these should not be stable, and indeed in the literature where applications are made (e.g., Zel'dovich and Raizer, 1966) the possibility has apparently not been considered before. We go on in §2.2, however, to argue that instability should arise whenever a condition is satisfied, equivalent to the presence of an entropy density which decreases outward. Physically the mechanism is identical with that responsible for convective instability in static stratified media when the temperature decreases in the upward direction. Analysis of the linearized fluid equations in §3 using the techniques developed by Bernstein and Book (1978) and Book (1978) confirms the existence of the instability and yields both the space and time dependence of the perturbations in closed form.

As was previously noted by Bernstein and Book (1978) and Book and Bernstein (1978), the usual definition of stability is inadequate when applied to nonsteady states, since the time dependence of the perturbations is in general not exponential. It is appropriate to call a mode stable (unstable) if the ratio of the perturbation amplitude to that of the basic state vanishes (diverges) as $t \to \infty$. We find that in the present case the ratio is in general finite, but can be made arbitrarily large in "unstable" systems by choice of sufficiently large mode number .

A brief discussion of the results in §4 concludes the paper.

1. The basic state

1.1 Derivation of the equations

We start with the equations of ideal hydrodynamics which in Lagrangian variables take the form

$$\dot{\rho} + \underset{\sim}{v} \cdot \nabla\rho = 0, \tag{2.1a}$$

$$\rho\dot{\underset{\sim}{v}} + \nabla p = 0 \tag{2.1b}$$

and

$$(p\rho^{-\gamma})^{\cdot} = 0 \tag{2.1c}$$

Here dots denote time derivatives and γ in (2.1c) is the ratio of specific heats.

In a spherically symmetric system, (2.1a,b) become

$$\dot{\rho} + \rho R^{-2} \frac{\partial}{\partial R} (R^2 u) = 0, \tag{2.2a}$$

and

$$\rho\dot{u} + \frac{\partial p}{\partial R} = 0. \tag{2.2b}$$

For a motion of the type known as homogeneous (or uniform) self-similar flow (Sedov 1959), the position R at time t of a fluid element whose position at t = 0 was r is required to satisfy

$$R = r\, f(t), \tag{2.3}$$

where $f(0) = 1$ and $\dot{f}(0) = 0$. The continuity equation (2.2a) then yields

$$\rho(r,t) = \rho_o(r) f^{-3}, \tag{2.4}$$

and hence from the adiabatic law (2.1c)

$$p(r,t) = p_o(r) f^{-3\gamma} = s(r)\rho_o^{\gamma}\ f^{-3\gamma} \tag{2.5}$$

with the entropy function s arbitrary. If we choose the initial density profile in the form

$$\rho_o(r) = \hat{\rho}(1-r^2/r_o^2)^{\kappa} \tag{2.6}$$

where $\hat{\rho}$ and κ are constants, then it follows from (2.1b) that

$$p_o(r) = \hat{p}(1-r^2/r_o^2)^{\kappa+1}, \tag{2.7}$$

and f must satisfy

$$\ddot{f}\, f^{3\gamma-2} = \frac{2(\kappa+1)\hat{p}}{\hat{\rho}\, r_o^2} = \tau^{-2}. \tag{2.8}$$

We will use the separation constant τ, the initial radius r_o, and the peak mass density $\hat{\rho}$ to rescale t, r and ρ, respectively. In these reduced variables we have

$$\rho_o = (1-r^2)^{\kappa}, \tag{2.9}$$

$$p_o = \frac{(1-r^2)^{\kappa+1}}{2(\kappa+1)}, \tag{2.10}$$

and

$$\ddot{f}\, f^{3\gamma-2} = 1. \tag{2.11}$$

A quadrature can be performed on (2.11), with the result

$$\dot{f}^2 = 2\ln f \tag{2.12}$$

if $\gamma = 1$, and

$$\dot{f}^2 = \frac{2}{\alpha}(1 - f^{-\alpha}) \tag{2.13}$$

otherwise, where $\alpha = 3(\gamma-1)$. If $\gamma = 5/3$, (2.13) can be integrated directly to give $f(t) = \pm(1 + t^2)^{\frac{1}{2}}$. For other values of γ the solution is most conveniently found by numerical means. At large $|t|$ when

$\ddot{f} \rightarrow 0$, the motion asymptotically approaches free streaming. As a function of the anisentropicity parameter κ, the solutions include the cases of uniform density and quadratic pressure, $\kappa = 0$, and uniform entropy density, $\kappa = 1/(\gamma-1)$. If we exclude singular density profiles, κ is restricted to $0 \leq \kappa < \infty$. The other parameter is γ, which must lie in the range $1 \leq \gamma < \infty$.

2.2. Physical mechanism for instability

At time t a small volume ΔV of fluid initially located at radius r contains a mass $\Delta m = \rho_o(r)\Delta V f^{-3}$, subjected to a pressure $p = (1-r^2)^{\kappa+1}/[2(\kappa+1)f^{3\gamma}]$. Consider two such fluid elements initially at radii r_1 and $r_2 > r_1$, whose volumes are related by

$$\Delta V_2/\Delta V_1 = (p_1/p_2)^{1/\gamma} = [(1-r_1^2)/(1-r_2^2)]^{(\kappa+1)/\gamma}. \tag{2.14}$$

The compressional energy associated with these elements is

$$E_p = (p_1\Delta V_1 + p_2\Delta V_2)/(\gamma-1). \tag{2.15}$$

Their kinetic energy calculated from the expansion or contraction of the sphere is

$$E_k = \tfrac{1}{2}(\Delta m_1 u_1^2 + \Delta m_2 u_2^2). \tag{2.16}$$

Because the state is nonsteady ($\ddot{f} \neq 0$), the elements are subject to an effective gravitational acceleration $g = r\ddot{f}$, and therefore have

a gravitational potential energy

$$E_g = \tfrac{1}{2} f \ddot{f} \, (\Delta m_1 r_1^{\,2} + \Delta m_2 r_2^{\,2}) \, . \tag{2.17}$$

Now let the two fluid elements interchange positions. By (2.14), they contract or expand so as to satisfy local pressure balance after the interchange. Furthermore, the work done in compressing one is just balanced by that done by the expansion of the other, so the compressional energy E_p' afterwards is equal to E_p. The net change in energy is then

$$\begin{aligned}
\delta E &= E_p' + E_k' + E_g' - E_p - E_k - E_g \\
&= \tfrac{1}{2}\,[\Delta m_1 u_2^{\,2} + \Delta m_2 u_1^{\,2} - \Delta m_1 u_1^{\,2} - \Delta m_2 u_2^{\,2}] \\
&\quad + \tfrac{1}{2} f\ddot{f}\,[\Delta m_1 r_2^{\,2} + \Delta m_2 r_1^{\,2} - \Delta m_1 r_1^{\,2} - \Delta m_2 r_2^{\,2}] \\
&= \tfrac{1}{2}(\dot{f}^2 + f\ddot{f})(r_2^{\,2} - r_1^{\,2})(\Delta m_1 - \Delta m_2) \, .
\end{aligned} \tag{2.18}$$

The first and second factors in the last member of (2.18) are strictly positive. The third factor, on the other hand, is proportional to

$$(1 - r_1^{\,2})^{\varkappa} - (1 - r_2^{\,2})^{\varkappa} \left(\frac{1-r_1^{\,2}}{1-r_2^{\,2}}\right)^{\frac{\varkappa+1}{\gamma}} \tag{2.19}$$

$$= (1-r_1^{\,2}) \left[1 - \left(\frac{1-r_1^{\,2}}{1-r_2^{\,2}}\right)^{\frac{\varkappa+1-\varkappa\gamma}{\gamma}} \right] \tag{2.19}$$

This expression is negative for $\kappa < 1/(\gamma-1)$. In this case, therefore, the interchange <u>reduces</u> the total system energy. We thus anticipate that an instability will set in, characterized by "overturning" of the profiles, such as is typically seen in convective or thermal instabilities of static media (Landau and Lifshitz, 1959).

When $\kappa < 1/(\gamma-1)$, $\delta E > 0$, in which case no instability should arise. The marginal case just corresponds to isentropic--more properly, homentropic--states. By (2.5), the entropy function s satisfies

$$s(r) = (1-r^2)^{\kappa+1-\kappa\gamma}. \tag{2.20}$$

The stable (unstable) case corresponds to outward increasing (decreasing) s(r). Evidently the physical picture here is analogous to that arising in connection with instabilities driven by a temperature inversion in media with a stratified density. Destabilization takes place owing to the buoyancy experienced by fluid elements in the nonuniform inertial gravity field. It is therefore purely a consequence of the nonsteady character of the basic state.

3. Analysis of the perturbed equations

We follow Bernstein and Book (1978) and Book (1978) in obtaining linearized equations for the development of a small perturbation about the solutions of §2. For simplicity we consider only expanding states ($t > 0$). The perturbed displacement $\underset{\sim}{\xi}$ satisfies the linearized form of (2.1b),

$$\rho\ddot{\underset{\sim}{\xi}} + \rho_1 \ddot{\underset{\sim}{R}} = - \nabla_{\underset{\sim}{R}} p_1 + \nabla_{\underset{\sim}{R}} \underset{\sim}{\xi} \cdot \nabla_{\underset{\sim}{R}} p. \tag{3.1}$$

Substituting for $\underset{\sim}{R}$ from (2.3), the perturbed density from

$$\rho_1 = - \rho \nabla_{\underset{\sim}{R}} \cdot \underset{\sim}{\xi}, \tag{3.2}$$

and the perturbed pressure from

$$p_1 = \frac{\partial p}{\partial \rho} \rho_1 = - \frac{\gamma(1-r^2)^{1+\varkappa(1-\gamma)}}{2(\varkappa+1)} \rho^{\gamma} \nabla_{\underset{\sim}{R}} \cdot \underset{\sim}{\xi}, \tag{3.3}$$

we obtain (writing $\nabla = \nabla_{\underset{\sim}{r}}$)

$$f^{\alpha+2} \ddot{\underset{\sim}{\xi}} = \frac{\gamma(1-r^2)}{2(\varkappa+1)} \nabla(\nabla\cdot\underset{\sim}{\xi}) - (\gamma-1) \underset{\sim}{r} \nabla \cdot \underset{\sim}{\xi} - \nabla\underset{\sim}{\xi} \cdot \underset{\sim}{r} . \tag{3.4}$$

Letting $\sigma = \nabla \cdot \underset{\sim}{\xi}$ and $\underset{\sim}{\omega} = \nabla \times \underset{\sim}{\xi}$, we have, on taking the divergence and curl of (3.4),

$$f^{\alpha+2} \ddot{\sigma} = \frac{\gamma}{2(\varkappa+1)} \nabla \cdot [(1-r^2)\nabla\sigma] - \gamma\underset{\sim}{r} \cdot \nabla\sigma$$

$$- (3\gamma-2)\sigma + \underset{\sim}{r} \cdot \nabla \times \underset{\sim}{\omega} \tag{3.5}$$

and

$$f^{\alpha+2} \ddot{\underset{\sim}{\omega}} = \left(1 - \frac{\gamma\varkappa}{\varkappa+1}\right)\nabla\sigma \times \underset{\sim}{r} + \underset{\sim}{\omega} . \tag{3.6}$$

We look for solutions of (3.5)-(3.6), assuming $\underset{\sim}{\xi}$ is separable into a

product of a function of position and a factor T(t) satisfying

$$f^{\alpha+2}\ddot{T} = \mu T, \tag{3.7}$$

μ constant. We further assume separation of the angular and radial dependence by writing

$$\sigma(\underset{\sim}{r}) = \sigma(r)\, Y_{\ell m}(\theta,\phi). \tag{3.8}$$

In (3.5) $\underset{\sim}{\omega}$ appears only in the form $\underset{\sim}{r} \cdot \nabla \times \underset{\sim}{\omega}$, for which an expression in terms of σ can be derived from (3.6) and (3.8):

$$(\mu-1)\underset{\sim}{r} \cdot \nabla \times \underset{\sim}{\omega} = \frac{1-(\gamma-1)\varkappa}{\varkappa+1}\left[2\underset{\sim}{r}\cdot\nabla\sigma + \underset{\sim}{r}\underset{\sim}{r}:\nabla\nabla\sigma - r^2\nabla^2\sigma\right]$$

$$= \sigma[1-(\gamma-1)\varkappa]\,\ell(\ell+1)/(\varkappa+1). \tag{3.9}$$

Substitution in (3.5) yields a second-order equation for the radial factor $\sigma(r)$,

$$\frac{\gamma}{2(\varkappa+1)}\left[\frac{1-r^2}{r^2}\frac{d}{dr}\left(r^2\frac{d\sigma}{dr}\right) - \frac{1-r^2}{r^2}\,\ell(\ell+1)\sigma - 2r\frac{d\sigma}{dr}\right]$$

$$-\gamma r\frac{d\sigma}{dr} + \left[\frac{(\varkappa+1-\varkappa\gamma)\ell(\ell+1)}{(\mu-1)(\varkappa+1)} - 3\gamma - \mu + 2\right]\sigma = 0. \tag{3.10}$$

Rewriting this equation by means of the substitutions $\sigma = r^{\ell}y$ and $x = r^2$, we obtain the hypergeometric equation

$$x(1-x)y'' + [c - (a+b+1)x]\, y' - aby = 0, \quad (3.11)$$

where

$$\left.\begin{matrix} a \\ b \end{matrix}\right\} = \tfrac{1}{2}\left\{\varkappa + \ell + 5/2 \pm [(\varkappa + \ell + 5/2)^2 - 4K]^{\frac{1}{2}}\right\}, \quad (3.12a,b)$$

$$c = \ell + 3/2. \quad (3.12c)$$

Here

$$K = \frac{(\varkappa+2)\ell}{2} + \frac{\varkappa+1}{2\gamma}\left[\mu - 2 + 3\gamma - \frac{\ell(\ell+1)(\varkappa+1) - \varkappa\gamma}{(\mu-1)(\varkappa+1)}\right]. \quad (3.13)$$

The solution of (3.11) which is finite at the origin is the hypergeometric function $y = {}_2F_1(a,b;c;x)$.

The boundary condition is found from the requirement that the perturbed pressure vanish on the interface with the vacuum. Since the unperturbed pressure already vanishes there, it follows from (3.3) that y need only be finite at $x = 1$. The analytic continuation formulas (e.g., Abramowitz and Stegun 1968) contain a term that diverges as $(1-x)^{-(\varkappa+1)}$ unless a or b is a nonpositive integer. Thus we must have

$$-n = \tfrac{1}{2}\{\varkappa + \ell + 5/2 - [(\varkappa + \ell + 5/2)^2 - 4K]^{\frac{1}{2}}\}, \quad (3.14)$$

$n = 0, 1, 2, \cdots$. Treating n and K as analytic functions of μ and differentiating (3.14) yields $\partial n/\partial\mu < 0$. Hence the fastest growth (largest $\mu > 1$) corresponds to the smallest value of n, viz., $n = 0$, which implies $K = 0$. Solving for μ, we finally obtain the dispersion relation

$$\mu-1 = -\frac{[\gamma\ell(\varkappa+2)+(\varkappa+1)(3\gamma-1)]}{2(\varkappa+1)}$$

$$\pm\frac{\{[\gamma\ell(\varkappa+2)+(\varkappa+1)(3\gamma-1)]^2+4\ell(\ell+1)(\varkappa+1-\varkappa\gamma)\}^{\frac{1}{2}}}{2(\varkappa+1)} \quad . \qquad (3.15)$$

For the upper branch, $\mu>1$ for all $\ell>0$, provided $\varkappa<1/(\gamma-1)$. The latter is precisely the condition derived from the energetic argument of §2.2.

Returning to (3.7), we find that, provided $\gamma>1$, the time dependence can likewise be expressed in terms of hypergeometric functions in the form (Bernstein and Book, 1978).

$$T(t) = T(0)\,\mathcal{F}(t) + \dot{T}(0)\,\mathcal{G}(t), \qquad (3.16)$$

Here,

$$\mathcal{F}(t) = {}_2F_1[\tfrac{1}{4} + \tfrac{2+\Delta}{4\alpha},\ \tfrac{1}{4} + \tfrac{2-\Delta}{4\alpha};\ \tfrac{1}{2};\ 1-f^{-\alpha}], \qquad (3.17a)$$

$$\mathcal{G}(t) = [\tfrac{2}{\alpha}(1-f^{-\alpha})]^{\frac{1}{2}}\,{}_2F_1[\tfrac{3}{4} + \tfrac{2+\Delta}{4\alpha},\ \tfrac{3}{4} + \tfrac{2-\Delta}{4\alpha};\ \tfrac{3}{2};\ 1-f^{-\alpha}], \qquad (3.17b)$$

and $\Delta = [(\alpha+2)^2 - 8\mu\alpha]^{\frac{1}{2}}$. For late times (large f), the analytic continuation formulas yield

$$\mathcal{F}(t) \sim \frac{\Gamma(1/2)\Gamma(1/\alpha)\ f}{\Gamma[1/4+(2+\Delta)/4\alpha]\Gamma[1/4+(2-\Delta)/4\alpha]}, \qquad (3.18a)$$

$$\mathcal{G}(t) \sim \frac{(2/\alpha)^{\frac{1}{2}}\ \Gamma(3/2)\Gamma(1/\alpha)\ f}{\Gamma[3/4+(2+\Delta)/4\alpha]\Gamma[3/4+(2-\Delta)/4\alpha]}\quad . \qquad (3.18b)$$

The numerical coefficients in (3.18) grow exponentially with μ for $\mu >> 1$. The case of $\gamma=1$ is very similar, except that confluent hypergeometric functions replace ${}_2F_1$, as observed by Bernstein and Book (1978) and Book and Bernstein (1978), and (3.18) is replaced by expressions proportional to $f(\ell n\, f)^{(\mu-1)/2}$.

4. Discussion

We have seen on energetic grounds that a certain class of spherical ideal gas expansions can be expected to be unstable whenever the gradient of the entropy density decreases with increasing r. Detailed analysis of the linear perturbations about these nonsteady basic states confirms this prediction, provided we appropriately generalize the usual definition of instability. Somewhat surprisingly, the solutions fall out exactly without recourse to numerical approximations, owing to the separability of the linearized equations.

As noted in §1, what matters in determining the stability of a time-dependent motion is the relative size of the perturbations. By (3.18), the latter vary asymptotically like the unperturbed radius. At early times, however, when $\ddot{f}$ differs substantially from zero, the perturbations can be amplified dramatically. If $\mu >> 1$, they grow approximately exponentially for $t \leqslant 1$, experiencing $\sim \mu^{\frac{1}{2}}$ e-foldings. The total amplification and the time required to approach the asymptotic state in which they "freeze out" both increase with μ. As $\gamma \to 1$, both the total amplification and the time required to approach saturation diverge (Bernstein and Book, 1978). Since μ increases with increasing ℓ, decreasing κ, and decreasing γ, all of these trends tend to enhance instability.

Note that as $\ell \to \infty$, μ diverges. This implies that the problem is not mathematically well-posed. In any real physical system, dissipative phenomena related to viscosity, thermal conduction, radiation, etc., set an upper limit on the mode number for which the ideal fluid model is valid. For shorter-wavelength disturbances than this, not only the detailed perturbation analysis, but the whole physical picture must be drastically different.

The perturbations studied here have radial dependence which peaks at $r = r_o$. They therefore should be most readily observable as an enhanced mixing or turbulence near the periphery of the expanding cloud. Since the instability is controlled by the sign of the entropy gradient, it seems likely that the nonlinear limit to which it tends is characterized by $ds/dr \geq 0$, $0 \leq r \leq r_o$. Whether this limit is actually attained is beyond the scope of the present work.

Another, perhaps more important, question remains unanswered. Uniform self-similar motion is an analytically convenient model used to approximate real flows. To what extent is the instability treated here associated with the latter, to what extent an artifact of the model? The present paper can of course provide no rigorous answer. Nonetheless, it seems physically plausible that for flows sufficiently close to uniform expansion, the results of the present analysis must be applicable. Even for nonuniform motions, either analytically or numerically described, the energetic argument of §2.2 can be employed and should again correctly predict the presence or absence of instability.

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